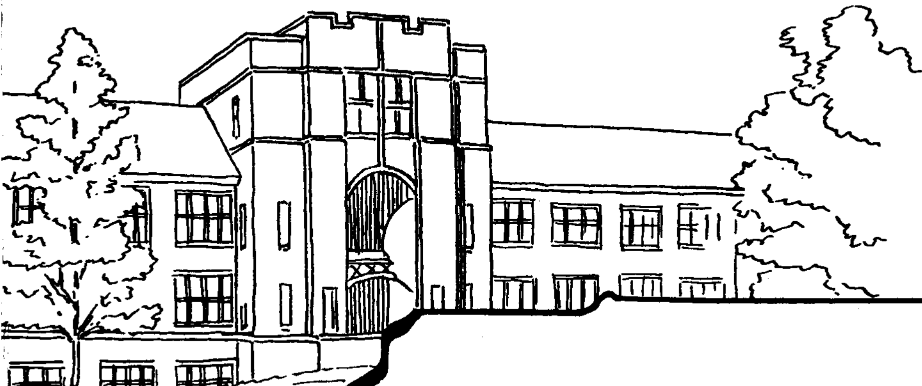
Control Theory Group



THE UNIVERSITY OF TENNESSEE

DEPARTMENT OF ELECTRICAL ENGINEERING

ANNUAL STATUS REPORT ON

NASA RESEARCH GRANT NO. NsG-351-Sup-1

FOR THE PERIOD OF

April 1, 1964, to April 30, 1965

N65-85938
(ACCESSION NUMBER)
11
(PAGES)
CR63635
(NASA CR OR TMX OR AD NUMBER)
(THRU)
N
(CODE)
(CATEGORY)
FACILITY FORM 602

prepared for
National Aeronautics and Space Administration
by

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June 15, 1965

ANNUAL STATUS REPORT ON

NASA RESEARCH GRANT NO. NsG-351-Sup-1

This annual status report covers the thirteen-month period from April 1, 1964, to April 30, 1965. The report is divided into sections as follows:

Research Work Completed
Research in Progress
Publication List
Research Personnel
Annual Cost Summary

Research work Completed

The following projects of control theory investigation have been completed during the thirteen-month period from April 1, 1964, to April 30, 1965.

I. Design of Minimum Energy and Minimum Time-Weighted Energy Discrete-Data Control Systems:

In recent years considerable effort has been expended on the optimum design of discrete-data control systems, especially on deadbeat controls.

For systems without saturation, Kalman and Bertram presented a very elegant method. Kalman also proposed a method for saturating time-optimal control.

It is well known that for non-saturating time-optimal control an n-th order system can be brought from an initial state to a desired final state in n sampling periods or less. But for saturating control the required number of sampling periods is, in general, greater than n. Furthermore, the control is not unique. A unique control can be obtained by imposing an additional constraint, for example, the so-called minimum energy control.

In many practical problems, the system output is required to be error-free after a finite period $NT > nT$, but the time-optimal response is not necessary. In this paper a procedure is developed for designing the minimum sum-of-input-squares, conventionally called minimum energy, discrete-data control of an n-th order continuous plant preceded by a zero-order hold. That is we want

$$E = \sum_{k=1}^{N} \left| m(k) \right|^2 = \text{minimum}$$

where m(k) is the system input during the period $(\overline{k-1}T, kT)$. The system is required to reach the origin of the state space from a given initial state in N sampling periods, with $N > n$. An important matrix, called the derived matrix has been developed, which expresses the relationship between the canonical vectors. The minimum energy control depends on the derived matrix only, which is in a very simple form and is easy to implement. The method is not restricted by the order of the system. The importance of this type of control has been discussed.

Extension of the above technique to the design of time-weighted energy has been made. Here the quantity to be minimized is

$$E = \sum_{k=1}^{N} d(k) \left| m(kt) \right|^2.$$

The advantage of time-weighted minimum energy control is that by a suitable choice of d(k), it is possible to approach the terminal state more gradually and yet have a faster rise time.

The results of this investigation were published as items No. 1 and No. 2 in the Publication List.

II. Minimum Energy Design of Discrete-Data Control Systems.

In any deadbeat design of discrete-data control systems where $N > n$, including the methods mentioned in Section I, the system is open-loop between every N sampling periods, therefore any load disturbance effect is uncompensated until the end of each N sampling periods. In many control problems, the deadbeat feature is really not necessary, all one asks for are good transient responses and accuracy.

Methods for the design of discrete-data control for a desired transient response via the dominant pole concept have been proposed. These methods require that the sampling frequency be high compared to the frequency of the dominant closed-loop poles, limiting the usefulness of the methods.

Another method is to employ the statistical design approach. However, this method leaves little room for settling time adjustment.

A new approach has been developed for designing a discrete-data feedback control system. First, the number of sampling periods, N, required for settling is chosen. Then a control law is designed for producing a deadbeat response with minimum plant input energy, that is,

$$E = T \sum_{k=1}^{N} \left| m(k) \right|^2 = \text{minimum}$$

where m(k) is the plant input during the period $(\overline{k-1}T, kT)$. Only m(1) is ever applied to the plant input. At each sampling instant the plant state, which is to be used as the new initial state is identified, and a new m(1) corresponding to this initial state is applied to plant input. Since the system is closed-loop at each sampling instant, disturbances are immediately compensated. System output response obtained in this way is very satisfactory, although it is not deadbeat. The risetime can be controlled by adjusting N, which is done by examining the eigenvalues of the closed-loop system.

The avoidance of multiple order poles of the closed-loop transfer function, associated with the non-deadbeat response, also prevents the system from having infinite differential sensitivity with respect to plant parameter variation.

The result of this investigation will be published as item No. 3 in the Publication List, to be presented at International Federation of Automatic Control Tokyo Symposium on August 25-28, 1965, Tokyo, Japan.

III. Geometrical Interpretations and Graphical Solution of Minimum Energy Control.

The mathematical approach used in considering the minimal energy problem for discrete systems (Section I above) is capable of giving not only very compact solutions but also sheds light on the problem of saturation. Furthermore, the problem of energy conservation is most easily considered in the canonical vector space.

<u>Geometrical Approach to the Saturation Problem</u>.

Previous approaches to the saturation problem in discrete systems include programming methods and more recently the use of functional analysis. With the forcing function limited in absolute value by saturation only a certain region of the state space (or the canonical vector space) can be taken to the origin in N sampling periods or less. This well known region is termed the set Γ_N. A subregion inside Γ_N

is the set M_N which is defined as the set of states that can be taken to the origin in N sampling periods with an input sequence, considered to be unconstrained, that minimizes E but which in fact does not violate the saturation limits. A knowledge of this set is useful because we would like to know whether we are likely to violate the saturation limits if we use the simple linear design equations on a given disturbed initial state.

The set M_N is developed in the canonical vector space using only the derived matrix. For second order systems, a simple graphical procedure has been developed which gives M_N and also allows the estimation, within graphical accuracy, of the input sequence.

Several interesting properties of M_N have been obtained which are not limited to second order systems. These properties can be used to avoid the saturation problem.

Geometrical Approach to Energy Conservation in Linear Systems.

While time-optimal regulation is of importance in many cases, it is of great practical and theoretical interest to know if it is possible, and if so how and under what conditions, to trade minimum time of regulation and the minimum energy associated with this time for a longer regulation time and less energy.

This research investigates the possibilities and limitations of reducing energy consumption by increasing the number of sampling periods, N, available for the system to reach equilibrum. It is found that for a given initial state the energy required for regulation is reduced if N is increased. The basis of the evaluation depends upon the shape and size of the set of all initial states that can be taken to the origin in a given time, N sampling periods, and a given energy E, with E the minimum energy consumption. These sets are termed the minimum energy regions, $M_N(E)$. The rate of decrease of the energy with increasing N can be obtained directly from these sets, and thus the best trade-off for a particular situation is clearly displayed.

An example has been tried for a second order system to demonstrate the simplicity of the method. The analog computer has been used in a novel way to generate the minimum energy regions, which are Lissajous figures in the "canonical" vector space; and the patterns are used directly to obtain a plot of the minimum energy consumptions as a function of the settling time.

The result of this research will be published as item No. 4 in the Publication List of this report.

IV. Optimum Recovery of The Thrust Transient of a Rocket Engine from The Distorted Measurement Record:

The accurate measurement of transient thrust characteristics is an important problem. In aerospace engineering the study of thrust build-up and tail-off requires the measurement of the thrust transient produced by rocket engines. This measurement is complicated by the fact that the bandwidth of the measuring instrument is much smaller than the bandwidth of the measured signal. The problem exists because a mechanical thrust stand with a large inertia is the device that is used to measure the rapid changing rocket thrust. Under this condition the output of the thrust stand is no longer a faithful replica of its input.

Geothert proposed a numerical method to recover the original signal from the distorted signal. This method involves the use of a trigonometric series and digital computation. The method is very complicated and is subject to truncation error of the trigonometric series. A very large number of terms must be taken to produce a satisfactory recovery.

Sprouse and McGregor proposed an analog method which makes use of a closed loop correcting system in cascade with the thrust stand. The parameters of the closed-loop system are adjusted to give a satisfactory thrust recovery. However, no criterion is given for this adjustment. Moreover, the simulation of the closed-loop system, which has a rather involved transfer function in the feedback loop, is subject seriously to analog computer noise.

An investigation has been made to devise a better thrust transient recovery scheme. The method developed uses an open-loop correcting network in cascade with the thrust stand output. This open-loop network is simulated by a small analog computer. The network parameters are so adjusted that the overall system, including both the thrust stand and the correcting network, has a maximally flat frequency response. It has been found that increasing the order of the correcting system does not give appreciable improvement to the result. Comparison has been made between all three methods. It is found that this last scheme gives much better results and requires less hardware.

The result of this investigation was published as item No. 5 in the Publication List of this report.

V. Pole Sensitivity of Feedback Systems:

Engineers have long been using the term "sensitivity" to denote a quantitative measure of the variation of one system parameter due to variation of another system parameter. Different mathematical definitions of sensitivity have been proposed by Bode, Truxal, Ur and Chang for different purposes. However, it is found that these definitions are not always convenient for practical engineering uses. For instance, does a sensitivity equal to zero really imply the complete insensitiveness of a system? And, does a measure of infinite sensitivity truly mean an infinite change of one system parameter with respect to a finite variation of another parameter?

For engineering purposes, a good definition of sensitivity measure should have the following qualities:

1. significance
2. reliability
3. convenience in use

The object of this investigation is to review the various definitions of "sensitivity," and to propose more satisfactory mathematical definitions of the "pole sensitivity of a feedback system" (Hereafter called "pole sensitivity" for simplicity) for engineering applications.

It is found that the form of Bode-Truxal's sensitivity function, is troublesome when used as a measure of a pole sensitivity under the following conditions:

(i) When the pole of the closed-loop transfer function, whose sensitivity is desired, has multiplicity greater than one.

(ii) When the pole of the closed-loop transfer function, whose sensitivity is desired, is at the origin.

(iii) When the pole, or zero, of the open-loop transfer function, with respect to which the closed-loop sensitivity is desired, is at the origin.

The first two conditions have been shown to yield infinite sensitivity while the third condition yields zero zensitivity.

From the result of this investigation, it is proposed that the sensitivity of a closed-loop pole s_j with respect to the open loop gain K be

$$S_K^{s_j} = \frac{(\Delta s_j)^m}{\frac{\Delta K}{K}} \quad ,$$

and the sensitivity of the closed-loop pole to open loop poles or zeros be

$$S_X^{s_j} = \frac{(\Delta s_j)^m}{\Delta x}$$

where m is the multiplicity of the closed-loop pole.

A convenient method of evaluating the proposed sensitivity functions has been developed.

The result of this investigation has been published as item No. 6 in the Publication List of this report.

VI. Classification of Adaptive Control Systems:

With the progress being made in space, and other industrial technologies, there is a growing need for automatic control systems which are capable of changing their own parameters in order to remain efficient in spite of large changes in environment and system characteristics.

In the literature on adaptive control systems one finds that different research groups have used their own terms and definitions like passive adaptive systems, active adaptive systems, computing type adaptive systems, self-organizing systems, self-optimizing systems, learning systems, pretaught systems, etc. Many of these terms overlap each other and sometimes the same term has been used by various groups for systems which belong to different categories.

It is felt, therefore, that there is a great need for a precise and clear-cut scheme of classifications of different types of systems which can be used for standard definitions. This general classification should include all the possible types.

A scheme of classification of adaptive control systems has been developed, and it has been shown that a large variety of adaptive, "self-optimizing" and "learning" control systems can be fitted into this scheme.

The scheme divides all adaptive systems into three classes, namely, basic adaptive systems, static adaptive systems, and dynamic adaptive systems.

It may be pointed out that in the literature on the so-called "learning" systems, there is some controversy on the proper use of the word learning. Systems belonging to the dynamic adaptive class could be considered as really of the learning type, but probably it is desirable not to use the word "learning" due to the confusion it may create.

The result of this study has been published as item No. 7 in the Publication List of this report.

VII. Research on the Nonlinear Problems Involved in the Steering of Spacecrafts Using On-off Control:

Research has been done to investigate the effects of time delay and nonlinear characteristics such as deadband, hysteresis, etc., in an on-off control system involving more than one nonlinearity. Systems of this type are difficult to handle by conventional techniques for nonlinear analysis.

A new method, called the rate-diagram method, was used for this investigation. Problems of stability, limit cycles, transient response, duty cycle, etc., were investigated. It was found that the rate diagram technique is very useful for an inertial plant, such as a space vehicle in free space.

The result of this study is being prepared as item No. 8 in the Publication List.

Research in Progress

I. Process Identification:

It has been found that the orthogonal projection method of optimum estimation is most suitable for optimum data recovery involved in space technology. One major problem which hinders the effective use of this technique, is that the dynamics of the system which generated the random process must be known beforehand. This problem is further complicated when the process is time varying. Research is underway to develop a practical scheme which identifies the dynamics which generate the random process.

II. Continued Investigation on Discrete-Data Control Systems:

Although many "advances" have been made with the recent theoretical tools of Linear, Nonlinear and Dynamic programming these programming techniques are far from the universal panacea. They intrinsically fail to give insight into many problems; there is a pressing need for synthesis and design techniques which themselves suggest improvements to existing control systems and which suggest novel and simpler hardware for implementation of the theory.

While the Minimum Energy Control of PAM plants has been investigated in some detail, research is continuing to develop the practical application of the theory. Other cost criteria and the problem of constraints on movement of certain variables are also under investigation.

III. Low Sensitivity Control Systems:

The development of a design procedure for low sensitivity control systems is being continued. This development uses the sensitivity functions proposed by Hung. It is felt that the developed method will be very convenient for systems having multiple poles, and for feedback systems having a zero or a pole of the open-loop function at the origin.

IV. Use of Rate Diagram Technique for Nonlinear Problems:

The "rate diagram" method currently under preliminary investigation will be developed further if it is found promising. It is felt this technique may be very helpful in handling a system containing more than one nonlinearity.

V. Critical Investigation of Error Analysis Techniques used for Trajectory Error Analysis:

The advent of the space age has brought forth a requirement for the development of techniques for the guidance and control of spacecraft in flight. To satisfy a part of this requirement, techniques of error analysis must be developed.

An error analysis technique is a procedure which can be used to predict the state error vector at a fixed time as a function of the source errors. The "transformation technique" and the "adjoint method" are the generally accepted error analysis techniques used in practice.

The basic idea for the transformation technique of error analysis was conceived and developed by Norton in the early fifties. The technique was later extended by Braham and Skidmore.

The method of adjoint functions was first used by Bliss in ballistic perturbation theory in the early forties. In recent years, the method of adjoint systems has become a standard mathematical procedure for solving linear, time-varying, ordinary differential equations. Laning and Battin describe applications of adjoint techniques to the study of random inputs to linear systems. Rogers and Connolly discuss the use of adjoint techniques with the analog computer. The adjoint method has also proven useful in optimum control theory, sensitivity analysis, celestial mechanics, etc.

In error analysis the adjoint method has been investigated by Hung, Coulter, Peske and Ward, and Battin.

There exist several different points of view as to which error analysis technique should be used in a practical situation. Research is underway on a critcal comparison between these two techniques and to investigate which technique should be applied to practical problems.

Publication List

1. Design of Minimum Energy Discrete-Data Control Systems, A. M. Revington and J. C. Hung, NASA-CR-60604, January, 1965.

2. Design of Time-Weighted Minimum Energy Discrete-Data Control Systems, A. M. Revington and J. C. Hung, NASA-CR-60512, February, 1965.

3. Minimum Energy Design of Discrete-Data Control Systems, J. C. Hung and A. M. Revington, invited for presentation at International Federation of Automatic Control Tokyo Symposium, August 25-28, 1965, Tokyo, Japan; will appear in Proc. IFAC Tokyo Symposium.

4. Geometrical Interpretation and Graphical Solution of Minimum Energy Time-Optimal Discrete-Data Control Systems, in preparation.

5. Optimum Correction of Thrust Transient Measurements, J. E. Irby and J. C. Hung, Scientific Report No. 6, Control Theory Group, Electrical Engineering Dept., The University of Tennessee, January, 1965.

6. A New Measure of Pole Sensitivity of Feedback Systems, J. C. Hung, NASA-CR-58239, June, 1964.

7. Adaptive Control Systems, A General Classification Scheme, N. K. Sinha, Technical Note No. 2, Control Theory Group, Electrical Engineering Dept., The University of Tennessee, October, 1964.

8. Nonlinear Analysis of Spacecraft Steering using On-off Control, in preparation.

Research Personnel

1.	James C. Hung, D.Sc.	Principal Investigator
2.	N. K. Sinha, Ph.D.	Senior Investigator
3.	J. David Irwin, M.E.E. (Doctoral Candidate)	Junior Investigator
4.	Anthony M. Revington, M.E.E. (Doctoral Student)	Junior Investigator
5.	C. F. Chen, B.S. in E.E.	Research Assistant (Part Time)
6.	John E. Irby, B.S. in E.E.	Research Assistant (Part Time)

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